# A Metaphysics of Incompleteness:

## Understanding Russell's Paradox

By Dennis J. Darland February 2020

#### Abstract.

I reason that incompleteness lies at the heart Russell's Paradox.

#### **The Argument**

Attempts to "solve" Russell's Paradox are attempts to admit all valid arguments while excluding invalid ones. In 1903 and 1904 Russell followed Frege's strategy. "His Plan of attack can be broken down into three parts. First, analyze as carefully as possible the conditions giving rise to the contradictions; this involves getting the most general versions possible of the arguments leading to the paradoxes. Second, isolate the formal properties of the propositional functions that lead to the paradoxes. Third, turn those formal properties into definitions by postulating that a proposition determines a class unless it satisfies these formal properties." (<u>The Collected Papers of Bertrand Russell</u>, volume 4, pp. xxi-xxii)

First, I am going to take a realist stance regarding logical and mathematical fact. Second, I will assume an infinity of objects. From these two, by Cantor's Theorem, there is no largest Cardinal number. But the number of sentences in logic is denumerable. So, there are necessarily more facts of logic or math than there are sentences of logic. Therefore, any valid logic must be incomplete. There must be classes which cannot be determined in any such manner by formal properties. Either some classes will be excluded or there will be contradictions. It is no coincidence that Russell's strategy seemed "reasonable, since all the paradoxes known to Russell involved some version of Cantor's diagonal argument." (CPBR 4, p. xxii)

### Conclusion

There are ways to avoid the paradoxes, but they necessarily exclude some valid conclusions.