An Analysis of Quantum Phenomena
in the Relative Product Theory of Propositional Attitudes

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Abstract.

I account for Quantum Phenomena within my Relative Product Theory of Propositional Attitudes.

Notation.

I use notation from Principia Mathematica to *56 [hereafter PM] as much as possible. Backward apostrophe is PM's notation for *function application* (PM *30.01). sin`x would, in usual algebra, be written sin(x). In "On Substitution", i is a function from objects to ideas of the objects. n is a function from objects to their names. The *relative product* (PM *34.01) of two relations R and S is written R|S and defined by

$x(R|S)y$ =df. $(\exists z)(R(x, z) \bullet S(z, y))$.

Cnv is the relation between a relation and its converse (PM *31.01), so Cnv`R is the converse of the relation R.

I introduce a special case of definite descriptions - *14.01

$((\iota\,\varphi)(S(f, \varphi))\ \{(\iota\,x)S(u, x), (\iota\,y)S(v, y)\}$ =df *14.01

$(\exists g)((z)(S(f, z) \equiv z = g)) \bullet (\exists c)((a)(S(u, a) \equiv a = c)) \bullet (\exists d)((b)(S(v, b) \equiv b = d)) \bullet g\{c,d\}$.

There are three entities described, (ιφ), (ιa) and (ιb). They may be taken to be the g, c, and d such that g is the relational object (predicate) such that f is uniquely related to it by the S relation. Please notice that this

$(\iota\ \varphi)(S(f, \varphi)$ is a definite description in predicate position – something not done in PM as far as I know, but it is defined in terms PM uses with predicates – quantification and identity (of g). c is the object such that u is uniquely related to it by the S relation, and d is the object such that v is uniquely related to it by the S relation. And g(c, d). Explained more straightforwardly (I hope), f is the idea of g, i.e. (S(f, g)) , and g is the only such entity. I am assuming S to be a many-one relation. That amounts to assuming our ideas to be unambiguous. Also, u is the idea of c, i.e. S(u, c); and c is the only object in that relation. Similarly, v is the idea of d and d is the only object in that relation. And the g relation holds between c and d.

In summary:

$(\exists\ g)\ (z)\ (S(f, z) \equiv (z = g))$

Assures that there is one and only one such z and it is g.

$(\exists\ c)\ (a)\ (S(u, a) \equiv (a = c))$

Assures that there is one and only one such a and it is c.

$(\exists\ d)\ (b)\ (S(v, b) \equiv (b = d))$

Assures that there is one and only one such b and it is d.
Note that this is PM *14.01 applied 3 times. f, u and v are values – being ideas that are determined previously in the overall statement. In each of the S(idea, object), should be regarded as a function of the object. S(idea, x) is φx in *14.01. The idea is regarded as fixed previously.

Then having required the existence and uniqueness of g, c, and d, we require in addition that g(c, d).

If the existence or uniqueness of any of the g, c, or d fail the statement is false. It is also false, in addition, if g(c, d) is not true.

It is perhaps easier to understand with

$((\iota\,\varphi)(S(f,\varphi)))\ \{(\iota\,x)S(u,x),\ (\iota\,y)S(v,y)\}\ \ =df$

$(\exists\, g)((\iota\,\varphi)(S(f,\varphi)) = g) \bullet (\exists\, c)((\iota\,x)S(u,x) = c) \bullet (\exists\, d)((\iota\,y)S(v,y) = d) \bullet g\{c,d\}$

Introduction – Modified Postulates for Quantum Mechanics.

I am trying to modify my theory to account for the postulates of quantum mechanics. I am using <u>Basic Quantum Mechanics</u> by Robert L. White as a reference, pages 29-31.

Postulate 1

This is my modification:

On this theory, for a actual occasion A at time t, there is a relation $P^{A,t}$ such that $(\exists y_n)(\exists x_n)x_n\, P^{A,t}\, y_n \equiv (\exists a_n)\, x_n = p^{A,t}{}_n\, y_n$ where $P^{A,t}$ is a quantum operator, each y_n is one of the functions out of the complete set of functions upon which the operator $P^{A,t}$ is meaningfully defined, and p_n is a number (perhaps complex). The solution y_n is an eigenfunction of $P^{A,t}$. These represent physical prehensions of the actual occasion A.

Postulate 2 – slightly modified – superscripts added

The only possible result of a measurement of the physical observable $p^{A,t}$ is one of the eigenvalues of the operator $P^{A,t}$

Postulate 3

For every dynamical system there exists a state function ψ which contains all the information that is known about the system.

Postulate 4

If we know ψ for any system at some particular time, then the evolution of ψ at all subsequent times is determined by

$$-\left(\frac{h}{2\pi i}\right) \partial\Psi/\partial t = H_{op}\,\psi$$

Where H_{op} is the operator associated with the Hamiltonian of the system.

Introduction.

On this theory, for an actual occasion A at time t, there is a relation $S^{A,t}$ involved in belief that is between ideas and the entities they are ideas of. There may, as an alternative, be physical events (relations $P^{A,t}$ to other actual occasions prehended), between impressions and entities they are impressions of. In Assertion there is a relation *R* involved between names (words) and the corresponding ideas or impressions. The P relation is given by physics. The R and S relations are taken as primitive here, but may be analyzed in the future,

In the case of physical prehensions,

$P^{A,t}(x_n, z_n)$ =df. at time t, x is an impression A has of physical prehension z_n. Physical Prehensions always follow the laws of physics. If there is such a case of involving $P^{A,t}$ and x^n then such z_n exists.

In the case of conceptual prehensions,

$S^{A,t}(x_n, y_n)$ =df. at time t, x_n is an idea A has of y_n
Conceptual prehensions may be faulty. $S^{A,t}$ and x_n may exist without any such y_n existing.

Words may be related to either physical or conceptual prehensions by R.

$R^{A,t}(w, x_n)$ =df. at time t, *w* is a name A has of the idea or impression x_n

On my view, the relations P, *R* and *S* are objective and cannot be assured to hold merely by the psychological will of a person. For an actual occasion A to have a belief, the actual occasion must form the relevant ideas or impressions which are involved in the belief, but in some cases, the *R* or *S* relations (or both) may fail to hold. P cannot fail, but may be conceptually misinterpreted. If the *S* relation fails to hold then what is formed is not a what I call a "full belief" since what is formed is unduly private and psychological. For this I introduce

$B_{private}(A, x, y, z)$

It is assumed that in such a case, *x*, *y*, *z* are all ideas or impressions formed by A. It is left open whether any of the ideas in question are ideas of entities which exist. But if, for example $S^{A,t}(x_n, a_n)$, then x_n is an idea of a_n, and it is assured thereby that a_n exists. One can have an idea that doesn't stand in the S relation to anything. It is in this sense alone that one can be said to "have an idea of something that doesn't exist." Similarly, if the *R* relation fails to hold, then an assertion of the belief is not possible.

$B_{private}$ is a primitive relation of an actual occasion to its ideas

In addition to R and S, I also take there to be a primitive psychological relation that may hold between an actual occasion's A's ideas or impressions – even if no corresponding objects exist (in the case of ideas). I call it $B_{private}$.

With the P, R and the S relations now in place, one can observe that the relative product of those relations, namely R|S and R|P are relations between a name (word) and an entity. That is because we have:

$x(R|S)y_n$ =df. $(\exists z_n)(R(x, z_n) \bullet S(z_n, y_n)$ and

$x(R|P)y_n$ =df. $(\exists z_n)(R(x, z_n) \bullet P(z_n, y_n)$

There may be objects for which an actual occasion A has no ideas or impressions, but in such a case A could not form singular beliefs about those specific objects. Also, there may be objects for which an actual occasion A has no words and thus A cannot make singular assertions about those objects. Now I have the following:

Full Belief defined.

$\mathrm{B}_{private}$ and S are primitives.

A believes that $\varphi(a, b)$ =df.

$(\exists x, y, z)\, B_{private}(A, \varphi_l, a_m, b_n) \bullet S^{A,t}(a_m, x)) \bullet S^{A,t}(b_n, y))\ ) \bullet S^{A,t}(\varphi_l, z)$

In the above, ϕ_l, a_m and b_n are A's ideas before his mind as so guaranteed to exist since A has formed the belief. x, y, and z are variables – there may be no values that satisfy the relations. $P^{A,t}$ may be substituted for $S^{A,t}$ above, except in that case there should be such values.

True Belief.

A believes truly that $\varphi(a, b)$ =df.

$B_{private}(A, \varphi_l, a_m, b_n) \bullet ((\iota z)(S^A(\varphi_l, z)))\{(\iota x)(S^A(a_m, x)), (\iota y)(S^A(b_n, y)\}$

In the above, ϕ_l, a_m and b_n are actual occasion A's ideas before his mind as so guaranteed to exist since he has formed the belief. The truth of the belief is given by a predicate and its arguments - all given by

definite descriptions. If the descriptions fail then there is no true belief. The definition of this form of definite descriptions is given in the section on notation above. Note: in $((\iota z)(S^A(\varphi_l, z)))\ \{(\iota x)(S^A(a_m, x)), (\iota y)(S^A(b_n, y)\}$
the described predicate is applied to two described objects. Again, $P^{A,t}$ may be substituted for $S^{A,t}$ above, except in that case there should be such values.

False Belief.

Actual occasion A believes falsely if actual occasion A believes but does not believe truly. Actual occasion A believing falsely is not the same as it being false that actual occasion A believes.

A believes falsely that *φ(a, b) =df.*

$B_{private}(A, \varphi_l, a_m, b_n) \bullet S^{A,t}(a_m, x)) \bullet S^{A,t}(b_n, y))\ ) \bullet S^{A,t}(\psi_l, z)$
$\bullet \sim ((\iota z)(S^{A,t}(\varphi_l, z)))\ \{(\iota x)(S^{A,t}(a_m, x)), (\iota y)(S^{A,t}(b_n, y)\}$

Again, $P^{A,t}$ may be substituted for $S^{A,t}$ above, except in that case there should be such values.

Assertion defined.

We are now ready to define assertion, which is distinct from belief. I take "saying" as a primitive relation between names (i.e. words).

A asserts that Φ *(a, b) =df.*

Says(A, Φ ,a, b) • ($\exists$w, u, v) R^{A} (Φ, w) • R^{A} (a, u) • R^{A} (b,v)

Φ , a, and b are words in the assertion – so they exist if actual occasion A made the assertion (said it). There will be w, u and v if the words have meaning to actual occasion A. If there are no such [ideas] w, u, or v then the existential quantifier fails and there is no assertion. Also, perhaps, the ideas must be limited to ones which make sense when so combined. That is beyond the scope of this paper and will not be considered. There may be no objects [corresponding to the ideas] with Cnv\`$S^{A,t}$ relations to w, u, or v. In that case, there still can be such an assertion. That is A may use words which correspond to ideas – but ideas to which no objects correspond. Yet, an assertion is made.

True Assertion.

A asserts truly that *Φ (a, b) =df.*
Says(A, Φ ,a, b) • ($\exists$ w, u, v) $R^{A,t}$ (Φ, w)) • $R^{A,t}$ (a, u)) • $R^{A,t}$ (b,v)
• ((ι z)($S^{A,t}$ (w, z))) {(ι x) $S^{A,t}$(u, x), (ι y) S^{A}, t(v, y)}

This is different. To be true, there must be objects with the Cnv\`$S^{A,t}$ relations to the ideas *w, u, and v*. We also need *((ι z)($S^{A,t}$ (w, z))) {(ι x) $S^{A,t}$(u, x), (ι y) $S^{A,t}$(v, y)}*. This was defined in the section on notation, and also implies that those objects exist. Note you can assert things that you do not believe. Honest assertion would add $B_{private}$. $P^{A,t}$ may be substituted for $S^{A,t}$ above.

The objects are specified by definite descriptions, so the assertion makes sense (although false) if the objects do not exist. Actual occasion A does not assert truly if any of the objects *z, x*, or *y* [I am actually speaking of the corresponding descriptions] do not exist, but actual occasion A does assert, by the previous definition. There cannot be a true assertion without *z, x* or *y*. If any the ideas *w, u*, or *v* do not exist then there is no assertion – some of A's words, *Φ ,a, or b* have no meaning for A.

False Assertion.

Actual occasion A asserts falsely if actual occasion A asserts but does not assert truly. Actual occasion A asserting falsely is not the same as it being false that actual occasion A asserts.

A asserts falsely that Φ *(a, b) =df.*
Says(A, Φ *,a, b)* $\bullet$ $(\exists w, u, v)\, R^{A,t}(\Phi, w)) \bullet R^{A,t}(a, u)) \bullet R^{A,t}(b,v)$
$\bullet \sim ((\iota z)(S^{A,t}(w, z)))\,\{(\iota x)\, S^{A,t}(u, x), (\iota y)\, S^{A,t}(v, y)\}$

This requires A to make the statement and have ideas of his words. Otherwise there is not even a false assertion. The assertion is false if there are not objects corresponding to any of the ideas or the corresponding relation does not hold among them. Again, $P^{A,t}$ may be substituted for $S^{A,t}$ above.